

Universality in the relaxation dynamics of the composed black-hole-charged-massive-scalar-field system: The role of quantum Schwinger discharge

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The quasinormal resonance spectrum $\{\omega_n(\mu, q, M, Q)\}_{n=0}^{n=\infty}$ of charged massive scalar fields in the charged Reissner-Nordström black-hole spacetime is studied *analytically* in the large-coupling regime $qQ \gg M\mu$ (here $\{\mu, q\}$ are respectively the mass and charge coupling constant of the field, and $\{M, Q\}$ are respectively the mass and electric charge of the black hole). This physical system provides a striking illustration for the validity of the universal relaxation bound $\tau \times T \geq \hbar/\pi$ in black-hole physics (here $\tau \equiv 1/\Im\omega_0$ is the characteristic relaxation time of the composed black-hole-scalar-field system, and T is the Bekenstein-Hawking temperature of the black hole). In particular, it is shown that the relaxation dynamics of charged massive scalar fields in the charged Reissner-Nordström black-hole spacetime may *saturate* this quantum time-times-temperature inequality. Interestingly, we prove that potential violations of the bound by light scalar fields are excluded by the Schwinger-type pair-production mechanism (a vacuum polarization effect), a *quantum* phenomenon which restricts the physical parameters of the composed black-hole-charged-field system to the regime $qQ \ll M^2\mu^2/\hbar$.

I. INTRODUCTION

Wheeler's celebrated conjecture that black holes are bald [1, 2] has played a key role in black-hole physics over the last four decades [3–8]. This ‘no-hair’ conjecture asserts that static matter fields cannot be supported in the exterior regions of asymptotically flat black-hole spacetimes [1–8].

The no-hair conjecture [1, 2] thus suggests a simple and universal picture in which matter fields that propagate in the exterior spacetime regions of newly born black holes would eventually be scattered away to infinity or be absorbed into the central black hole. This suggested scenario is supported by the fact [9] that the dynamics of fundamental fields in black-hole spacetimes are characterized by ringdown (*damped*) oscillations of the form $e^{-i\omega t}$ [10–12]. These decaying oscillations, which are known as the quasinormal modes of the composed black-hole-field system, are characterized by an infinite spectrum of complex resonant frequencies $\{\omega_n\}_{n=0}^{n=\infty}$ [13].

In accord with the spirit of the no-hair conjecture, these decaying modes characterize the relaxation dynamics of composed black-hole-fundamental-field systems. In particular, the damped quasinormal oscillations reflect the gradual decay of the matter fields (the ‘hair’) in the exterior spacetime regions of newly born black holes [14, 15].

It should be emphasized, however, that existing no-hair theorems [1–8] say nothing about the *timescale* it takes for a newly born black hole to become bald. This characteristic relaxation timescale, τ_{relax} , is determined by the fundamental (least damped) black-hole quasinormal resonance:

$$\tau_{\text{relax}} \equiv 1/\Im\omega_0 . \quad (1)$$

An interesting question naturally arises: How short can the relaxation time (1) be?

A remarkably simple answer to this fundamental question was suggested in [16]: using standard ideas from information theory and thermodynamics, it was argued in [16] that the characteristic relaxation time of a perturbed thermodynamic system should be bounded from below by the time-times-temperature (TTT) inequality [17]

$$\tau_{\text{relax}} \times T \geq \frac{\hbar}{\pi k_B} , \quad (2)$$

where T is the system temperature [18]. In particular, in the context of black-hole perturbation theory, it was argued in [16] that the imaginary part of the fundamental black-hole quasinormal resonance should be bounded from above according to the simple relation [see Eqs. (1) and (2)]

$$\Im\omega_0 \leq \pi T_{\text{BH}} , \quad (3)$$

where T_{BH} is the black-hole temperature.

Interestingly, it was demonstrated numerically in [16] that the quasinormal resonances of spinning Kerr black holes indeed conform to the suggested upper bound (3). Moreover, it was proved analytically [19] (see also [20]) that rapidly-rotating (near-extremal) Kerr black holes may actually *saturate* the bound. Namely, their fundamental resonances are characterized by the simple asymptotic relation [21]

$$\Im\omega_0 \rightarrow \pi T_{\text{BH}} \quad \text{as} \quad T_{\text{BH}} \rightarrow 0. \quad (4)$$

II. CHARGED SCALAR FIELDS AND SCHWINGER-TYPE PAIR PRODUCTION

The relaxation dynamics of charged matter fields in a newly born *charged* Reissner-Nordström (RN) black-hole spacetime [22, 23] was studied analytically in [24, 25]. In particular, it was shown that the fundamental quasinormal resonances of *massless* ($\mu = 0$) charged scalar fields in the charged black-hole spacetime are characterized by the simple relation [25]

$$\Im\omega_0 = \pi T_{\text{BH}} [1 + O(qQ/l^2)] \quad \text{for} \quad \{\mu = 0 \text{ and } l \ll qQ \ll l^2\}, \quad (5)$$

where T_{BH} is the RN black-hole temperature [see Eq. (42) below]. Here q, l , and Q are respectively the charge coupling constant of the field, the spherical harmonic index of the field mode [see Eq. (14) below], and the electric charge of the black hole [26]. A similar result was later obtained in [27] for charged massless scalar fields in the regime $qQ \gg (l+1)^2$. Most recently, Ref. [28] extended this result to higher orders in the small quantity $(l+1)^2/qQ$, finding

$$\Im\omega_0 = \pi T_{\text{BH}} \left(1 + \pi T_{\text{BH}} \frac{r_+ - 3r_-}{4q^2 Q^2} \right) \quad \text{for} \quad \{\mu = 0 \text{ and } qQ \gg (l+1)^2\}, \quad (6)$$

where $r_{\pm} = M \pm (M^2 - Q^2)^{1/2}$ are the horizon radii of the RN black hole (here M is the black-hole mass).

Interestingly, the result (6) suggests a violation of the universal relaxation bound (2) [or equivalently, a violation of the black-hole quasinormal bound (3)] for charged RN black holes in the regime [29]

$$Q/M < \frac{\sqrt{3}}{2}. \quad (7)$$

In particular, *if* the gravitational collapse of a charged *massless* scalar field with $qQ \gg (l+1)^2$ could end up in the formation of a charged RN black hole with $Q/M < \sqrt{3}/2$, then the characteristic relaxation time [see Eqs. (1) and (6)]

$$\tau_{\text{relax}} = \frac{1}{\pi T_{\text{BH}}} \times \left(1 - \pi T_{\text{BH}} \frac{r_+ - 3r_-}{4q^2 Q^2} \right) < \frac{1}{\pi T_{\text{BH}}} \quad \text{for} \quad \{\mu = 0, \text{ and } qQ \gg (l+1)^2\} \quad (8)$$

of the newly born charged black hole would violate the universal relaxation bound (2) [30].

One naturally wonders: Is there a physical mechanism which can prevent the apparent violation (8) of the universal relaxation bound? It seems that no classical effect can prevent this violation. However, as we shall show below, the Schwinger-type pair-production mechanism [31–34], a purely *quantum* effect, ensures the validity of the (*quantum*) universal relaxation bound (2).

In particular, vacuum polarization effects set an upper bound on the electric field strength of charged RN black holes [31–34]: $E_+ < E_c \equiv \mu^2/q\hbar$, where $E_+ = Q/r_+^2$ is the electric field at the black-hole horizon. In fact, it was shown in [34] that the Schwinger-type pair production mechanism becomes effective (that is, with pair production probability of almost 100%) already at $E \simeq 0.03E_c \ll E_c$. Thus, the quantum Schwinger discharge of charged RN black holes (the vacuum polarization effect) sets an upper bound on the electric field strengths of the black holes: $Q/r_+^2 \ll \mu^2/q\hbar$, or equivalently

$$qQ \ll \mu^2 r_+^2. \quad (9)$$

The inequality (9) implies, in particular, that the charged *massless* scalar fields studied in [25, 27, 28] are physically unacceptable in the *quantum* regime.

In particular, taking cognizance of the fact that the suggested universal relaxation bound (2) is intrinsically a *quantum* phenomenon [35], one realizes that a self-consistent test of the bound validity in the context of charged gravitational collapse must include *massive* charged fields which respect the *quantum* [36] inequality (9).

The main goal of the present paper is thus to test the bound validity in a self-consistent manner, that is, in the physically acceptable regime (9). To that end, we shall study below the relaxation dynamics of a newly born charged RN black hole. In particular, we shall analyze the quasinormal resonance spectrum (the characteristic *damped* oscillations) of charged *massive* perturbations fields [see Eq. (9)] in the background of the newly born charged black hole.

III. DESCRIPTION OF THE SYSTEM

We analyze the dynamics of a charged massive scalar field linearly coupled to a Reissner-Nordström black hole of mass M and electric charged Q . The charged RN black-hole spacetime is described by the line element [37]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) , \quad (10)$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} . \quad (11)$$

The radii of the black-hole (event and inner) horizons,

$$r_{\pm} = M \pm (M^2 - Q^2)^{1/2} , \quad (12)$$

are determined by the zeros of $f(r)$.

The dynamics of a scalar field Ψ of mass μ and charge coupling constant q [38] in the black-hole spacetime is governed by the Klein-Gordon wave equation [39–41]

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu^2]\Psi = 0 , \quad (13)$$

where $A_\nu = -\delta_\nu^0 Q/r$ is the electromagnetic potential of the charged RN black hole.

It is convenient to decompose the scalar field Ψ in the form

$$\Psi(t, r, \theta, \phi) = \int \Sigma_{lm} e^{im\phi} S_{lm}(\theta) R_{lm}(r; \omega) e^{-i\omega t} d\omega , \quad (14)$$

where the integers l and m are the spherical harmonic index and the azimuthal harmonic index of the field mode, respectively [42]. Substituting the field decomposition (14) into the Klein-Gordon wave equation (13), one finds [39–41] that the radial and angular functions $[R(r)$ and $S(\theta)$, respectively] are determined by two differential equations of the confluent Heun type [43, 44]. These two equations are coupled by the angular eigenvalues (separation constants) $K_l = l(l+1)$ with $l \geq |m|$. The radial wave equation is given by [39–41]

$$\Delta \frac{d}{dr} \left(\Delta \frac{dR}{dr} \right) + UR = 0 , \quad (15)$$

where $\Delta = r^2 f(r)$, and

$$U = (\omega r^2 - qQr)^2 - \Delta[\mu^2 r^2 + l(l+1)] . \quad (16)$$

Defining the new radial function

$$\psi = rR , \quad (17)$$

and using the “tortoise” radial coordinate y , which is defined by the relation

$$dy = \frac{dr}{f(r)} , \quad (18)$$

one can write the radial equation (15) in the form of a Schrödinger-like wave equation

$$\frac{d^2 \psi}{dy^2} + V\psi = 0 . \quad (19)$$

Here

$$V = V(r; M, Q, \omega, q, \mu, l) = \left(\omega - \frac{qQ}{r} \right)^2 - \frac{f(r)H(r)}{r^2} \quad (20)$$

plays the role of an effective radial potential, where

$$H(r; M, Q, \mu, l) = \mu^2 r^2 + l(l+1) + \frac{2M}{r} - \frac{2Q^2}{r^2} . \quad (21)$$

The Schrödinger-like radial wave equation (19) is supplemented by the physically motivated boundary conditions of purely ingoing waves at the black-hole horizon and purely outgoing waves at spatial infinity [45]. That is,

$$\psi \sim \begin{cases} e^{-i(\omega - qQ/r_+)y} & \text{as } r \rightarrow r_+ \ (y \rightarrow -\infty) ; \\ y^{-iqQ} e^{i\sqrt{\omega^2 - \mu^2}y} & \text{as } r \rightarrow \infty \ (y \rightarrow \infty) . \end{cases} \quad (22)$$

These boundary conditions single out a *discrete* spectrum of complex eigenvalues $\{\omega_n(M, Q, \mu, q, l)\}_{n=0}^{\infty}$ which correspond to the quasinormal resonances of the charged massive scalar field in the charged RN black-hole spacetime. The main goal of the present paper is to determine *analytically* these characteristic resonances of the composed black-hole-scalar-field system in the physically acceptable regime (9).

IV. THE QUASINORMAL RESONANCE SPECTRUM OF THE COMPOSED BLACK-HOLE-CHARGED-FIELD SYSTEM

In the present section we shall analyze the resonance spectrum of the charged massive scalar fields in the charged RN black-hole spacetime. We shall focus on the large-coupling regime

$$qQ \gg \max\{\mu r_+, l + 1\} . \quad (23)$$

As we shall show below, in the regime (23) the radial potential (20) has the form of an effective potential barrier whose fundamental scattering resonances can be studied analytically using standard WKB methods [46–49].

In particular, in the large-coupling regime (23) the maximum r_0 of the potential barrier (20) is located in the vicinity of the black-hole horizon:

$$\frac{r_0 - r_+}{r_+ - r_-} \ll 1 . \quad (24)$$

To see this, it proves useful to introduce the dimensionless variables

$$x = \frac{r - r_+}{r} \quad ; \quad \tau = \frac{r_+ - r_-}{r_+} \quad ; \quad \varpi = \frac{\omega r_+}{qQ} - 1 , \quad (25)$$

in terms of which the effective radial potential (20) becomes

$$V(x; \varpi) = \left(\frac{qQ}{r_+}\right)^2 (x + \varpi)^2 - \frac{H(r_+)\tau}{r_+^2} x [1 + O(x/\tau)] . \quad (26)$$

Differentiating (26) with respect to the dimensionless radial coordinate x , one finds

$$x_0 + \varpi = \frac{H(r_+)\tau}{2q^2Q^2} \ll 1 \quad (27)$$

for the location x_0 of the maximum of the radial potential barrier [50].

As shown in [46, 47], the characteristic WKB resonance equation for the scattering resonances of the Schrödinger-like wave equation (19) can be written in the form

$$iK = n + \frac{1}{2} + \Lambda(n) + O[\Omega(n)] , \quad (28)$$

where [47]

$$K = \frac{V_0}{\sqrt{2V_0^{(2)}}} , \quad (29)$$

$$\Lambda(n) = \frac{1}{\sqrt{2V_0^{(2)}}} \left[\frac{1 + (2n + 1)^2}{32} \cdot \frac{V_0^{(4)}}{V_0^{(2)}} - \frac{28 + 60(2n + 1)^2}{1152} \cdot \left(\frac{V_0^{(3)}}{V_0^{(2)}}\right)^2 \right] , \quad (30)$$

and the cumbersome expression for the sub-leading correction term $\Omega(n)$ is given by Eq. (1.5b) of [47]. Here $V_0^{(k)} \equiv d^k V / dy^k$ are evaluated at the maximum $y = y_0$ of the effective potential barrier $V(y)$.

Taking cognizance of Eqs. (26), (27), (29), and (30), one finds

$$K = \frac{H(r_+)}{2qQ} \cdot \frac{\varpi - \frac{H(r_+)\tau}{4q^2Q^2}}{\frac{H(r_+)\tau}{2q^2Q^2} - \varpi}, \quad (31)$$

$$\Lambda(n) = -\frac{2qQ}{H(r_+)} \cdot \left(n + \frac{1}{2}\right)^2, \quad (32)$$

and

$$\Omega(n) = O[(qQ/H(r_+))^2(n + 1/2)^3]. \quad (33)$$

Note that the quantum constraint (9) implies that, in the regime $n \ll H(r_+)/qQ$ [51], the various terms that appear on the r.h.s of the WKB resonance equation (28) are characterized by the strong inequalities [51]

$$\frac{\Lambda(n)}{n + \frac{1}{2}} = O[(n + 1/2)qQ/H(r_+)] \ll 1 \quad \text{and} \quad \frac{\Omega(n)}{\Lambda(n)} = O[(n + 1/2)qQ/H(r_+)] \ll 1. \quad (34)$$

Substituting (31)-(33) into the WKB resonance equation (28), one finds

$$\varpi_R = \frac{H(r_+)\tau}{4q^2Q^2} \cdot \frac{H^2(r_+) + 2(2n+1)^2q^2Q^2[1 - (2n+1)qQ/H(r_+)]^2}{H^2(r_+) + (2n+1)^2q^2Q^2[1 - (2n+1)qQ/H(r_+)]^2} \cdot \{1 + O[(qQ/H(r_+))^2]\} \quad (35)$$

and

$$\varpi_I = -i \frac{H(r_+)\tau}{4q^2Q^2} \cdot \frac{(2n+1)qQH(r_+)[1 - (2n+1)qQ/H(r_+)]}{H^2(r_+) + (2n+1)^2q^2Q^2[1 - (2n+1)qQ/H(r_+)]^2} \cdot \{1 + O[(qQ/H(r_+))^2]\}. \quad (36)$$

Using the strong inequality [51]

$$\frac{qQ}{H(r_+)} \ll 1, \quad (37)$$

one can write the expressions (35)-(36) in the form

$$\varpi_R = \frac{H(r_+)\tau}{4q^2Q^2} \cdot \{1 + O[(qQ/H(r_+))^2]\} \quad (38)$$

and

$$\varpi_I = -i \frac{\tau}{2qQ} (n + 1/2) [1 - (2n+1)qQ/H(r_+)] \cdot \{1 + O[(qQ/H(r_+))^2]\}. \quad (39)$$

Finally, taking cognizance of Eq. (25), one finds

$$\omega_n = \frac{qQ}{r_+} + \frac{H(r_+)\tau}{4qQr_+} - i \frac{\tau}{2r_+} \cdot \left[n + \frac{1}{2} - \frac{2qQ}{H(r_+)} \cdot \left(n + \frac{1}{2} \right)^2 \right] \quad ; \quad n = 0, 1, 2, \dots \quad (40)$$

for the characteristic quasinormal resonances of the composed black-hole-charged-massive-scalar-field system. It is worth emphasizing again that the resonance spectrum (40) is valid in the physically acceptable regime (9).

V. SUMMARY AND DISCUSSION

We have analyzed the quasinormal resonance spectrum of charged massive scalar fields in the charged Reissner-Nordström black-hole spacetime. Our main goal in this paper was to test the validity of the suggested universal relaxation bound (2) in the context of black-hole physics (and, in particular, in the context of dynamical ‘hair’ shedding of newly born charged RN black holes).

It was first pointed out that charged *massless* fields in the regime $qQ \gg (l+1)^2$ can violate the relaxation bound (2). In particular, we have stressed the fact that, *if* the gravitational collapse of a charged massless scalar field in the large coupling regime $qQ \gg (l+1)^2$ could end up in the formation of a RN black hole with $Q/M < \sqrt{3}/2$, then the characteristic relaxation time (8) of the newly born charged black hole would violate the suggested universal relaxation bound (2).

However, given the fact that the universal relaxation bound (2) is intrinsically a *quantum* phenomenon [35], we have argued that a self-consistent test of the bound in the context of charged gravitational collapse must include *massive* charged fields which respect the *quantum* [36] inequality (9). This inequality is a direct consequence of the Schwinger discharge mechanism (a vacuum polarization effect), a quantum phenomenon which sets a bound on the physically allowed parameters of the composed black-hole-charged-massive-scalar-field system: $qQ \ll \mu^2 r_+^2 / \hbar$.

We have shown that the characteristic quasinormal resonances of the composed black-hole-charged-field system can be studied *analytically* in the regime

$$\max\{\mu r_+, l+1\} \ll qQ \ll \mu^2 r_+^2. \quad (41)$$

[The left inequality in (41) corresponds to the large coupling assumption (23), whereas the right inequality in (41) corresponds to the quantum constraint (9) imposed by the Schwinger-type pair production mechanism]. In particular, we have derived the resonance spectrum (40) in the regime (41).

Interestingly, as we shall now show, the quasinormal resonance spectrum (40) can be expressed in terms of the physical parameters of the composed black-hole-charged-field system: the black-hole temperature T_{BH} , the black-hole electrostatic potential Φ_+ , the black-hole electric field strength E_+ , and the critical electric field E_c for Schwinger-type pair production of the charged massive particles. In particular, using the relations

$$T_{\text{BH}} = \frac{r_+ - r_-}{4\pi r_+^2}, \quad \Phi_+ = \frac{Q}{r_+}, \quad E_+ = \frac{Q}{r_+^2}, \quad E_c = \frac{\mu^2}{q}, \quad (42)$$

one can write the resonance spectrum (40) in the form [52]:

$$\omega_n = q\Phi_+ + \pi T_{\text{BH}} \frac{E_c}{E_+} - i2\pi T_{\text{BH}} \cdot \left[n + \frac{1}{2} - 2 \frac{E_+}{E_c} \cdot \left(n + \frac{1}{2} \right)^2 \right] \quad ; \quad n = 0, 1, 2, \dots \quad (43)$$

Finally, we note that the quasinormal spectrum (43) implies the simple relation

$$\Im \omega_0 = \pi T_{\text{BH}} \left(1 - \frac{E_+}{E_c} \right) \quad (44)$$

for the fundamental (least damped) quasinormal resonance of the composed black-hole-charged-field system. This relation yields [53]

$$\tau_{\text{relax}} = \frac{\hbar}{\pi T_{\text{BH}}} \left(1 + \frac{E_+}{E_c} \right) > \frac{\hbar}{\pi T_{\text{BH}}} \quad (45)$$

for the characteristic relaxation time of the newly born charged black hole. This result [and, in particular, the last inequality in (45)] is in perfect harmony with the suggested universal relaxation bound (2).

Moreover, the relation (45) reflects the fact that, to leading order in the dimensionless small quantity E_+/E_c [53], the characteristic relaxation time $\tau_{\text{relax}} = \hbar/\pi T_{\text{BH}}[1 + O(E_+/E_c)]$ of the composed black-hole-field system in the physically acceptable regime (41) is *universal*, that is, independent of the charged-field parameters.

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- [52] Here we have used the relation [see Eq. (21)] $H(r_+) = \mu^2 r_+^2 \{1 + O[l(l+1)/\mu^2 r_+^2]\}$ in the regime $\mu r_+ \gg l+1$ [see Eq. (41)].
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